Advanced Optical Formulae

Phernell Walker, MBA, ABOM, LDO
Renowned International Speaker

PURE OPTICS

1

PURE OPTICS

Optometric Strategy | **Business Operations** | **Ophthalmic Education**

2

Phernell Walker, MBA, ABOM, LDO

- Principle | Pure Optics LLC
- Author | Pure Optics
- American Board of Opticianry | Board of Directors
- Pacific University College of Optometry | Past Adjunct Professor
- Master in Business Administration (MBA)
- Master in Ophthalmic Optics (ABOM)
- Licensed Dispensing Optician (WA-LDO)
- National Contact Lens Examiners Certified (NCLEC)

3

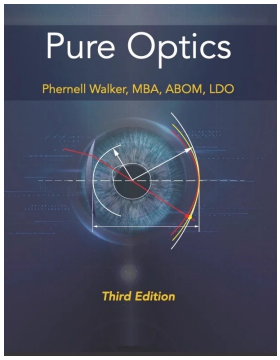
SCAN ME

Phernell Walker, MBA, ABOM, LDO

w: pure-optics.com

e: phernell@pure-optics.com

4



Resource

Pure Optics (2024)

Phernell Walker, MBA, ABOM, LDO

5

Objectives

- Geometric Optics
- Position of Wear Optics and Ophthalmic Lenses
- Optical Design Considerations
- Q & A

6

Lens Power

- Base Curve (front vertex power)
- Ocular Surface (anterior vertex power)
- Lens Thickness (measured in meters)
- Refractive Indice

7

Using Formulae' to Create Lens Appeal

Practical:

$$D_1 + D_2 + (t)\,(D_1)^2 / n = D_e$$

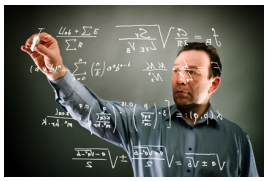
Exact:

$$[\, D_2 / 1 - (t/n)\,(D_2)\,] + D_1 = D_e$$

8

Variable Key

- D_1 = base curve
- D_2 = ocular curve
- t = thickness (M)
- n = refractive index
- D_e = total dioptric power
- 1 = constant



9

Lights Camera Action

A lens has a base curve of +9.00D, Ocular curve of -2.00D, 7 mm thick and is made of plastic 1.60n.

What is the lens power the patient will experience?

$$D_1 + D_2 + (t)\,(D_1)^2 / n = D_e$$

10

And there you have it!

Formula:

$D_1 + D_2 + (t)\,(D_1)^2 / n = D_e$

$+9.00 + -2.00 + (7\text{ mm})\,(9.00)^2 / 1.60 = D_e$

$+7.00 + (.007\text{ m})\,(81) / 1.60 = D_e$

$+7.00 + .567 / 1.60 = D_e$

$+7.00 + .35 = D_e$

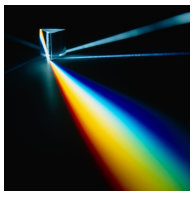
$+7.35 = D_e$

*Power experienced by the patient ignoring vertex distance

11

Resolving and Resulting Prism

- Rectangular (Resolving Prism)
- Polar Coordinate (Resultant)

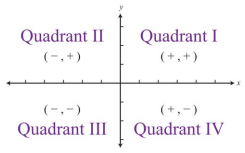


 12

12

Prism Rectangular Form

- B.I.
- B.O.
- B.U.
- B.D.
- Combination



13

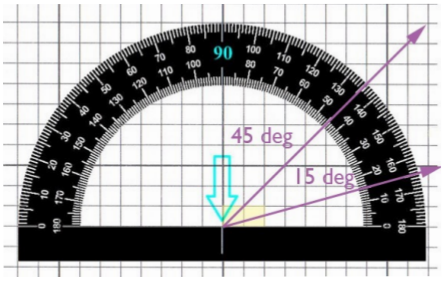
Rectangular Prism Notation

Polar coordinate prism notation indicates the base direction in degrees. There may be times when you will need to convert between rectangular and polar coordinate prism form.

This can be most useful when neutralizing lenses (determining the unknown power of a lens) with a lensometer.

14

Polar to Rectangular Prism Conversion



15

Polar Notation to Rectangular
$V = (D_e)$ (sine a)
$H = (D_e)$ (cosine a)

V = Vertical Coordinate
H = Horizontal Coordinate
D_e = Power of Prism

16

16

Polar to Rectangular Prism Conversion

Example:

Convert the following prescription from polar notation to rectangular notation:

O.D. +3.25 DS, 4 Prism, B.I. @ 045

17

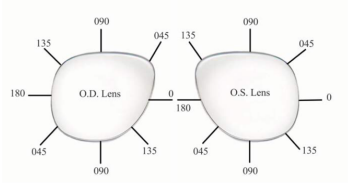
17

Solution:

OD: +3.25 DS, 4^{Δ} BI @ 045

V = (4.00) (.707)
H = (4.00) (.707)
V = 2.82
H = 2.82

OD: +3.25, 2.82^{Δ} B.U., 2.82^{Δ} B.I.Notice the rectangular coordinates for the right eye directly corresponds with the polar coordinate of 045 degrees (fig. 11-5).



18

18

Resultant Prism

When generating prescriptions and creating prism in an optical lab, it is important to know the exact location of the prism's base.

19

19

Rectangular to Polar Notation

$$\sqrt{P} = H^2 + V^2$$

$$\tan^{-1} a = V / H$$

where:

$\sqrt{P}$ = prism (square root of the prism)
V^2 = vertical coordinate
H^2 = horizontal coordinate

20

20

Convert Resultant into Polar Prism

Example:

OD +3.00 DS, 4 Prism B.I. & 2 Prism B.U. from rectangular to polar prism

21

21

$\sqrt{P} = H^2 + V^2$

$\sqrt{P} = 4^2 + 2^2$

$\sqrt{P} = 16 + 4$

$\sqrt{P} = 20$

$\sqrt{P} = 4.47\ D^{\Delta}$

$\tan^{-1}\ a = V / H$

$\tan^{-1}\ a = 2 / 4$

$\tan^{-1}\ a = .50$

Converted Rx: +3.00 DS, 4.47 D $^{\Delta}$ @ 27 degrees

22

Objects Viewed Are Skewed Through My Glasses

Rx:

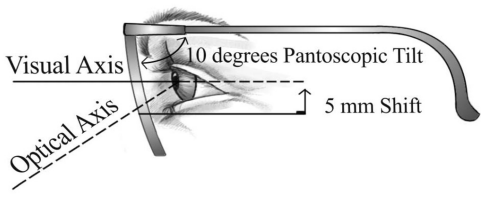
OD: -8.50 DS

OS: -8.50 DS

- Pantoscopic Tilt = 15 deg
- n = 1.498 (Cr-39)
- Vertex = 13 mm

23

Martin's Tilt Formula



24

Mastering Optics with Formulae

$S_{De} = S [1 + (\sin@)^2 / 2_n]$

$C_{De} = S_{De} (\tan@)^2$

Variable Key:

S_{De} = effective sphere power
S = sphere power
n = refractive index
C_{De} = effective cylinder

25

Step I: $S_{De} = S [1 + (\sin@)^2 / 2_n]$

$S_{De} = S [1 + (\sin@)^2 / 2_n$

$S_{De} = -8.50 [1 + (\sin15)^2 / 2 (1.498)]$

$S_{De} = -8.50 [1 + .06698 / 2.996]$

$S_{De} = (-8.50) (1 + .02235)$

$S_{De} = (-8.50) (1.02235)$

$S_{De} = -8.68$

26

Step II: $C_{De} = S_{De} (\tan@)^2$

$C_{De} = -8.68 (\tan15)^2$

$C_{De} = -8.68 (\tan15)^2$

$C_{De} = (-8.68) (\tan15)^2$

$C_{De} = (-8.68) (\tan15)^2$

$C_{De} = (-8.68) (0.2679)^2$

$C_{De} = (-8.68) (0.0717)$

$C_{De} = -0.62$

27

Final: Lens Tilt Resultant Rx

Original Rx:

OD: -8.50 DS
OS: -8.50 DS

Resultant Rx:

-8.68 -0.62 x 180

28

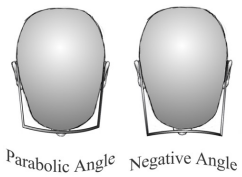
28

I Feel Nauseous

Rx:

OD: -6.00 -1.00 x 180
OS: -6.00 -1.00 x 180

- Parabolic Angle = 20 deg
- n = 1.70
- Vertex = 13 mm



29

29

Step I: Convert Rx to the 090th Meridian

Rx:

OD: -6.00 -1.00 x 180
OS: -6.00 -1.00 x 180

Rx:

OD: -7.00 +1.00 x 090
OS: -7.00 +1.00 x 090

30

30

Step I: $S_{De} = S\ [1 + (\sin@)^2 / 2_n]$

$S_{De} = S\ [1 + (\sin@)^2 / 2_n$

$S_{De} = -7.00\ [1 + (\sin 20)^2 / 2\ (1.70)]$

$S_{De} = -7.00\ [1 + 0.1169 / 3.40]$

$S_{De} = (-7.00)\ (1 + 0.034)$

$S_{De} = (-7.00)\ (1.034)$

$S_{De} = -7.238$

31

Step II: $C_{De} = S_{De}\ (\tan@)^2$

$C_{De} = 7.24\ (\tan 20)^2$

$C_{De} = 7.24\ (\tan 20)^2$

$C_{De} = (7.24)\ (\tan 20)^2$

$C_{De} = (7.24)\ (\tan 20)^2$

$C_{De} = (7.24)\ (0.36397)^2$

$C_{De} = (7.24)\ (0.13247)$

$C_{De} = +0.96$

32

Final: Lens Tilt x 180 Resultant Rx

Original Rx:

OD: -6.00 -1.00 x 180
OS: -6.00 -1.00 x 180

Original (Transposed) to 090th Meridian:
OD: -7.00 +1.00 x 090
OS: -7.00 +1.00 x 090

Resultant Effective Rx:

Answer: -7.24 +1.96 x 090

33

Magnification Factors

- Base Curve
- Dioptric Power
- Thickness
- Vertex Distance
- Refractive index

34

Magnification

Ms = 1 / (1- (t) (D_1)) / n
Mp = 1 / (1 - De (hm))
(Ms) (Mp) = Mt
(Mt -1) 100 = % of X

Ms = Shape Factor
Mp = Power Factor
Mt = Magnification Total

35

Practice Makes Perfect

Example:

A patient has the following prescription & fitting parameters:
- SV Lenses
- 1.66_n
- Vertex (h) = 13mm
- BC: +2.00D

What is the percentage of spectacle magnification?

O.D. -8.75 D.S. (thickness 5 mm)
O.S. -6.50 D.S. (thickness 4 mm)

36

Step I: Calculate Shape Factor

OD Lens Only:

MS = 1 / 1 – (t /n) (D_1)
MS = 1 / 1 – (.005/ 1.66) (2)
MS = 1 / 1 – (.003) (2)
MS = 1 / 1 – .006
MS = 1 / .994
MS = 1.006

37

Calculate Mx Power

O.D. Lens Only:

MP = 1 / 1 – (h) (D)
MP = 1 / 1 – (.013)(-8.75)
MP = 1 / 1 - .113
MP = 1 / .887
MP = 1.127

38

Step III: Step x Power

OD Lens Only:

SM = [(MS) (MP) -1] 100
SM = [(1.006) (1.127) - 1] 100
SM = [1.133-1] 100

Answer:

Spectacle Magnification = 13.30%

(Minus Lenses equal 13.30% demagnification)

39

Step I: Calculate Shape Power

O.S. Lens Only:

MS = 1 / 1 – (t /n) (D_1)
MS = 1 / 1 – (.004/ 1.66) (2)
MS = 1 / 1 – (.002) (2)
MS = 1 / 1 – .004
MS = 1 / .996
MS = 1.004

40

Step II: Calculate Power Factor

O.S. Lens Only:

MP = 1 / 1 – (h) (D)
MP = 1 / 1 – (.013) (-6.00)
MP = 1 / 1 - .078
MP = 1 / .922
MP = 1.084

41

Final Step

Multiply Shape & Power Factors

SM = [(SF) (PF) -1] 100
SM = [(1.004) (1.084) - 1] 100
SM = [1.088 - 1] 100
SM = 8.80% (minus power results in demagnification)

O.D. 13.30% demagnification
O.S. 8.80% demagnification
Difference = 4.50%.

42

Manipulating Plano Lenses

How can you manipulate the image size of a plano lens?

Plano lenses are considered “afocal” (dioptric power factor is unity), which means that a change in magnification can be created using the only the lens shape.

43

43

Magnification of Plano Lenses

$$Mt = -t\,(D_2) / 10_n$$

- Mt = Total magnification needed
- -t = thickness
- D_2 = ocular curve
- n = substrates refractive index

44

44

CC:	Blurry Vision
Location:	Distance Vision
Onset:	New Contacts
Severity:	6
Duration:	Constant
Mod. Fac:	None

Blurry Vision with New Soft Toric Contacts

45

Soft Toric Contact Lenses

Presenting Rx	SCOR
- Cyl: -8.50 -2.00 x 180	- CYL: -0.50 -0.75 x 140
+Cyl: -10.50+2.00 x 090	+ Cyl: -1.25 +0.75 x 050

46

Combining Cylinders

$C_e^2 = C_1^2 + C_2^2 + 2C_1 C_2 \cos 2y$

$S_e = (S_1 + S_2 + C_1 + C_2 - C) / 2$

$^{-1}\tan /2 = C_2 \sin 2y / C1 + C2 \cos 2y$

- Higher Axis Rx $= S_2\, C_2 \times a_2$
- Lower Axis Rx $= S_1\, C_1 \times a_1$
- $y = a_2 - a_1$

47

Calculate New Cylinder

$C_e^2 = C_1^2 + C_2^2 + 2C_1 C_2 \cos 2y$

$C_e^2 = 0.75^2 + 2^2 + 2\,(0.75)\,(2) \cos 2(40)$

$C_e^2 = 0.5625 + 4 + 1.50\,(2) \cos 80$

$C_e^2 = 0.5625 + 4 + 3\,(0.17)$

$C_e^2 = \sqrt{5.07}$

$C_e = +2.25$ *plus cylinder format

48

Calculate New Sphere

$S_e = (S_1 + S_2) + (C_1 + C_2 - C) / 2$

$S_e = (-10.50 + -1.25) + (0.75 + 2.00 - 2.25) / 2$

$S_e = -11.75 + -0.50/ 2$

$S_e = -11.75 + -0.25$

$S_e = -11.50$ *plus cylinder format

49

Calculate New Axis

$^{-1}tan /2 = C_2 \sin 2y / C_1 + C_2 \cos 2y$

$^{-1}tan /2 = 2.25 \sin 80 / 0.75 + 2 \cos 80$

$^{-1}tan /2 = 2.21/ 1.10$

$^{-1}tan /2 = 2.01$

$^{-1}tan /2 = 64$

Axis = 31.77 degrees of change

Add 32 degrees to a_1 **(32 + 50 = 082)**

50

Corrected Soft Toric Contacts

Presenting Rx	Corrected Rx
+ CYL: -10.50 +2.00 x 090	+ CYL: -11.50 +2.25 x 082
- CYL: -8.50 -2.00 x 180	- CYL: -9.25 -2.25 x 171

51

52